Full vectorial modal solutions for photonic crystal fibres by use of the finite element method

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Abstract: Modal solutions for photonic crystal fibres are presented by using a rigorous full vectorial finite element-based approach. The effective indices, spot-sizes, modal hybridness, beat lengths and group velocity dispersion values are determined for the quasi-TE and TM modes.

1. Introduction

Photonic crystal fibre (PCF), or 'holey fibre' is a micro-structured fibre, where arrays of holes run along the waveguide length, having the potential for more widely controllable fabrication parameters than for standard single mode fibre. Increasing interest is being shown in such PCFs for a range of applications in optical communications, sensing and signal processing. In a PCF, the number of holes and their sizes, shapes, orientations and placements as well as the dielectric material used can provide an additional degree of freedom which is not present in conventional fibre. A wide range of potential applications is anticipated, exploiting the ability to tailor the group velocity dispersion (GVD), the large spot-size for high power applications, and the smaller spot-size for improved nonlinear interactions, Raman amplification, Brillouin lasers, second harmonic generation, four-wave mixing, and creating polarization maintaining PCFs with higher modal birefringence and showing supercontinuum generation.

2. Theory

To date, most of the research into these fibres has had a strong experimental basis [1], which has recently been complemented by various modal solution approaches to their characterization, but mostly using scalar formulations or being limited to specific types of structures. The modal solution approach, based on the powerful finite-element method (FEM) is more flexible, can represent any arbitrary cross-section accurately and has been widely used to find the modal solutions of a wide range of optical waveguides [2]. The flexibility of the FEM to represent a cross-section of a holey fibre with arbitrary hole sizes and placements, makes it a powerful approach where many other simpler and semi-analytical approaches would be unsatisfactory. The optical modes in a high-index contrast PCF with two-dimensional optical confinement are also hybrid in nature, with all the six components of the **E** and **H** fields being present. To characterize accurately such fibres, a full-vectorial approach is necessary and such a **H**-field based full-vectorial approach [2] has recently been extended to study polarization issues in PCFs. Modal solutions for the fundamental TE and TM polarized modes have been obtained. As a result, polarization-dependent single mode operation, the variation in the spot-size, the modal field profiles, the modal hybridism, the birefringence, and the beat length have been calculated for these fibres.

3. Results

First a typical PCF structure with holes in a regular hexagonal honeycomb configuration is considered. For this regular array of holes of equal size, the hole diameter is taken as d (μm) with Λ (μm) as the period. A missing hole at the centre can guide the light as the core of a typical optical waveguide. Variations of the effective indices for the fundamental quasi-TE (H^y_{11}) modes with the operating wavelength are shown in Fig.1. In this case the hole diameter, d and period, Λ, are taken as 1.1 μm and 2.2 μm respectively. It can be noted that the guide index reduces gradually with the wavelength, as shown by a solid line, due to the material dispersion. However, it can be observed that the effective indices also reduces with the wavelength and its reduction is more than that of the silica index. This is due to the additional modal dispersion, with the mode being less confined when the operating wavelength is increased. The PCF structure considered above has a 90 degrees rotational symmetry and hence the TE and TM modes would be degenerate. The effective indices for the quasi-TM modes are the same as those of the quasi-TE modes and are not shown here. However, in the numerical simulation, since a two-fold symmetry is used with the electric and the magnetic field boundary conditions at the symmetry planes, these degenerate modes can be easily separated and interference between the degenerate modes is avoided.

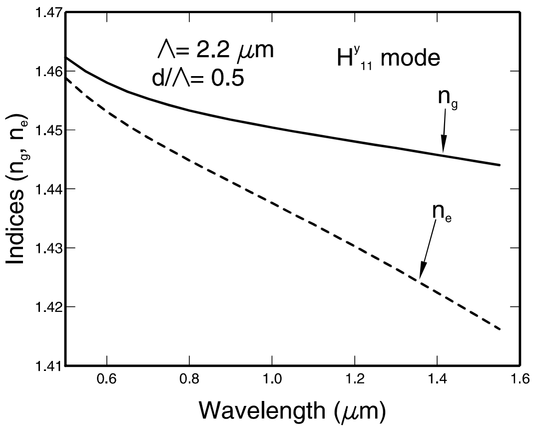


Fig.1. Variation of the silica index and the effective index with the wavelength for d/Λ = 0.5

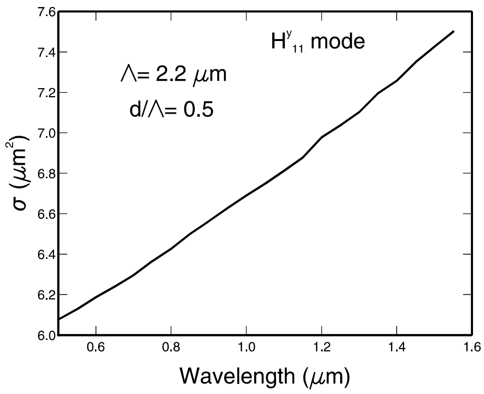
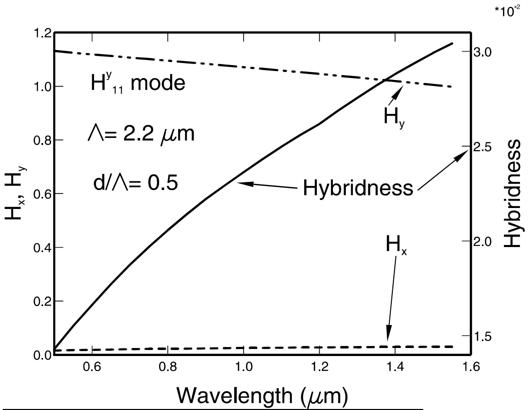


Fig.2. Variation of the spot-size with the operating wavelength for d/Λ = 0.5.

The variation of the spot-size with the wavelength, for the fundamental quasi-TE mode, is shown in Fig.2. In this case the spot-size has been defined as the area of the guide with more than 1/e of the maximum field intensity. It can be observed that as the operating wavelength is increased, the waveguide dimensions are reduced compared to the operating wavelength and the spot-size increases since the mode approaches its cutoff condition and so expands.

Modes in optical waveguides with two-dimensional confinement are not truly TE or TM, but hybrid in nature. For the quasi-TE (H^y_{mn}) modes, the H_y field component is dominant: however, the non-dominant field, H_x is not zero. The modal hybridness can be defined as the ratio of the non-dominant to the dominant field values. The variations of the vector field components for the quasi-TE mode are shown in Fig. 3. It can be observed that as the wavelength is increased, the dominant H_y field is reduced slightly and the non-dominant H_x field is increased. As a consequence the modal hybridness, which is the H_x/H_y ratio, also increases with the operating wavelength. The variation of the modal hybridness with the wavelength is also shown in Fig.3. It can be noted that the modal hybridness increases as the operating wavelength is increased, since in this case the modal confinement is reduced and the modal field is affected by the higher field strength at the silica/air interfaces.



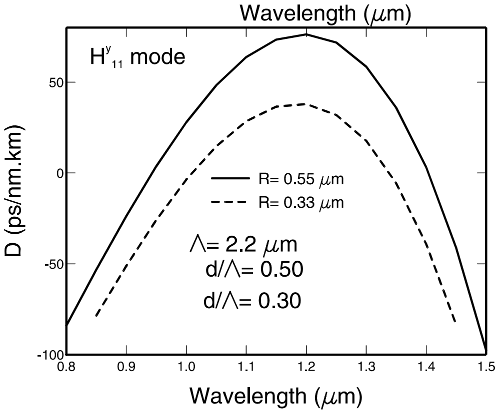


Fig. 3. Variation of the modal hybridness with the wavelengths for d/Λ = 0.5.

Fig. 4. Variations of the GVD with the wavelength for d/Λ = 0.5 and 0.3.

The group velocity dispersion (GVD) is one of the most important modal properties of an optical waveguide and this parameter can be defined as:

$$\mathrm{D}\,(\lambda) = -\frac{\lambda}{c}\,\frac{\partial^2 n_e}{\partial \lambda^2} \qquad (1)$$

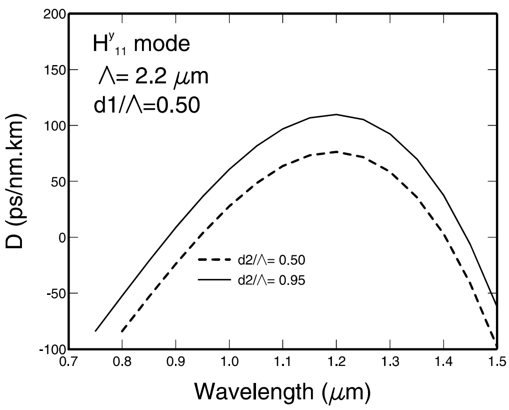


Fig. 5 Variation of the GVD with the operating wavelength for $d_2/\Lambda = 0.5$ and $d_2/\Lambda = 0.95$.

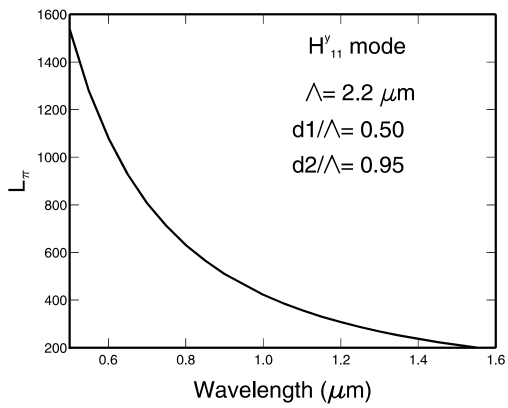


Fig. 6 Variation of the beat length with the operating wavelength for $d_1/\Lambda = 0.5$ and $d_2/\Lambda = 0.95$.

where n_e is the effective index of a given mode. Two different hole diameters have been considered, where *d* = 0.66 μm and 1.1 μm, yielding d/Λ ratios of 0.3 and 0.5, respectively. Variations of the GVD with the operating wavelength are shown in Fig. 4. It can be observed that a low anomalous GVD can be achieved over a given operating wavelength range, and similar adjustable GVD properties cannot be achieved in the design of the less flexible telecommunication grade SMF. It can be observed that by adjusting the diameters of the holes, then over the range of anomalous dispersion, its maximum value and the dispersion slopes can be adjusted.

The FEM approach is very versatile and it allows for the change of the position and size of any of the holes, as required. Next, the modal properties of a PCF are studied, where the 90 degree rotational symmetry does not exist. In this case the size of one of the air-holes, d_2, is different from the others. The variation of the GVD for this PCF structure is shown in Fig.5. For comparison, the GVD of the degenerate PCF, with $d_1=d_2$ = 1.1 μm, is also shown in this figure. Again, it can be observed that by adjusting the hole dimensions of a specific group of holes, the GVD properties can be modified.

The variation of the birefringent beat length with the operating wavelength is shown in Fig. 6. In this case the beat length, L_π may be defined as:

$$\mathrm{L}\pi = \frac{\pi}{\left|\beta_{\mathrm{y}} - \beta_x\right|} \qquad (2)$$

where β_y and β_x are the propagation constants of the corresponding quasi-TE and TM modes. It should be noted that as the operating wavelength increases, their effective indices (n_e) reduce but Δn_e increases, where Δn_e is the difference between their effective indices. It can be observed that with increasing operating wavelength, the beat length is reduced and this value is lower than 0.4 mm when the operating wavelength is more than 1.0 μm. It can be noted that the modal birefringence is much higher than that can be achieved simply by just adjusting the waveguide parameters of a SMF.

4. Conclusions

Important design parameters, such as the effective indices, spot-sizes, modal hybridness, beat length and the GVD are shown for both equal and unequal circular hole PCFs, by using the rigorous full vectorial finite element-based approach. The finite element method offers a versatile approach, which can represent any arbitrary-shaped PCF with arbitrary hole shapes, sizes, orientations and their placements. The variation of the GVD, an important optical parameter, shows the effect of the hole diameters and asymmetry and it may be possible to design a PCF with a specific GVD, or other optical properties, by adjusting the different fabrication parameters involved.

5. References

[1] J. C. Knight, T. A. Birks, P. St J. Russell, and D. M. Atkin, "All-silica single-mode optical fibre with photonic crystal cladding," Opt. Lett. **21**, 1547-1549 (1997).
[2] B. M. A. Rahman and J. B. Davies, "Finite-element solution of integrated optical waveguides," J. Lightwave Tech. **2**, 682-688 (1984).